Assignment: Module 11 Name: Hoyoung kim

Disclaimer: This is my work, not that of others

Total Score: 40

1. 10
2. 10
3. 10
4. 10

1. (10 pt) Use a 5th order interpolating polynomial to estimate 𝑦 at 𝑥 = 3.5. In your  
answer, please show the interpolating polynomial. You can use Python library to find  
the interpolating polynomial.  
x 0 1.8 5 6 8.2 9.2  
y 2.6 16.415 5.375 3.5 2.015 2.54

When using python’s polyfit, the result comes out as

5.74407918e-03, -1.73471191e-01, 1.99870979e+00, -1.04592064e+01

2.09771368e+01  2.60000000e+00

5.74407918e-03 x^5 -1.73471191e-01 x^4 + 1.99870979 x^3 -0.104592064 x^2

+ 20.9771368 x + 2.60000000

2. (10 pt) Given the data:  
x 1 2 2.5 3 4 5  
y=f(x) 0 5 7 6.5 2 0

a. Plot the data and estimate 𝑓(3.4) from the plot. (You can use excel to plot the  
data.)

b. Calculate 𝑓(3.4) using Newton Interpolating polynomial of order 1. (Do this by  
hand. Also, you only need two data point for order 1 interpolation. Make sure you  
pick the right points)

7-5/3-3.5

= 2/0.5= 4   
7 + 4(x-3.5)

f(3.4) = 7+4((3.4)-3.5) =6.6

c. Calculate 𝑓(3.4) using Newton Interpolating polynomial of order 5. (Do this by  
hand. Use finite divided differences to find the coefficients for each Newton  
interpolating polynomial.)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 0 |  |  |  |  |  |
|  |  | 5 |  |  |  |  |
| 2 | 5 |  | -0.66667 |  |  |  |
|  |  | 4 |  | -2.16667 |  |  |
| 2.5 | 7 |  | -5 |  | 1.166667 |  |
|  |  | -1 |  | 1.333333 |  | -0.28333 |
| 3 | 6.5 |  | -2.33333 |  | 0.033333 |  |
|  |  | -4.5 |  | 1.433333 |  |  |
| 4 | 2 |  | 1.25 |  |  |  |
|  |  | -2 |  |  |  |  |
| 5 | 0 |  |  |  |  |  |

f(x) = 0 + 5(𝑥 − 1) − 0.6667(𝑥 − 1)(𝑥 − 2) − 2.1666(𝑥 − 1)(𝑥 − 2)(𝑥 − 2.5)  
+1.1667(𝑥 − 1)(𝑥 − 2)(𝑥 − 2.5)(𝑥 − 3) − 0.2833(𝑥 − 1)(𝑥 − 2)(𝑥 − 2.5)(𝑥 − 3)(𝑥 − 4)

f(3.4) = 4.82

3. (10 pt) Given the data:  
x 1 2 3 5 6  
y=f(x) 4.75 4 5.25 19.75 36  
Estimate 𝑓(4) using Lagrange polynomials of order one and three. (Do this by hand.)  
f(x) = (x-5)/(3-5) \* 5.25 + (x-3)/(5-3) \* 19.75

f(4)= 2.625 +9.875 = 12.5

order3 =

4(𝑥 − 3)(𝑥 − 5)(𝑥 − 6)/ (2 − 3)(2 − 5)(2 − 6)+5.25(𝑥 − 2)(𝑥 − 5)(𝑥 − 6)/ (3 − 2)(3 − 5)(3 − 6)

+19.75(𝑥 − 2)(𝑥 − 3)(𝑥 − 6)/ (5 − 2)(5 − 3)(5 − 6)+36(𝑥 − 2)(𝑥 − 3)(𝑥 − 5)/ (6 − 2)(6 − 3)(6 − 5)

4(𝑥 − 3)(𝑥 − 5)(𝑥 − 6)/ -12+5.25(𝑥 − 2)(𝑥 − 5)(𝑥 − 6)/ 6

+19.75(𝑥 − 2)(𝑥 − 3)(𝑥 − 6)/ -6 +36(𝑥 − 2)(𝑥 − 3)(𝑥 − 5)/ 12

f(4) = 10

1. (10 pt) Given the data.  
   x 1 2 2.5 3 4 5  
   y 1 5 7 8 2 1  
   Fit these data with (a) cubic splines with natural end conditions, (b) cubic splines with  
   not-a-knot end conditions. Present comparative plots of 50 equally spaced  
   interpolation points over the domain 1 ≤ 𝑥 ≤ 5. You may use Python for this  
   problem. However, make sure you understand the definition of cubic spline and the  
   various end conditions.

A

when python is used to solve natural spline, which has errors due to round off error

Chart, line chart

Description automatically generated

B

when python is used to solve natural spline, which has less errors due to round off error

Chart, line chart

Description automatically generated